# RESEARCH INTERESTS

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# 1 Introduction & Overview

My main research interest is particle physics phenomenology, in particular phenomenological issues pertinent to QCD at low-energies. My current research activity is on the scalar mesons below 2 GeV. Understanding their status, in general, and their quark substructure, in particular, is an area of great current interest in theoretical and experimental particle physics [1] and lattice QCD [2]. Also, the role of scalar meson $\sigma$ in nuclear force has been a topic of intense research activity for a long time [3]. Recent works are given in refs. [4-27], and my contributions are described in some details in Sec. 2.

Investigating scalar mesons has three important aspects:

1. **Exploring low-energy QCD:**
   Scalars are important from the theoretical point of view because they are related to the Higgs bosons of QCD and induce chiral symmetry breaking, and therefore, are probes of the QCD vacuum. Scalars are also important from a phenomenological point of view, as they are crucial intermediate states in Goldstone boson interactions away from threshold; in a range of energy that is too high for a chiral perturbation theory framework, and too low in the context of the perturbative QCD. Moreover, the physics of scalar mesons has a great impact on our understanding of important issues in strong interactions such as the diquarks, glueballs, hybrids, violation of isospin, low energy hadron phenomenology, and final-state interaction of pseudoscalar mesons. The scalar mesons play substantial roles in several important experimental investigations such as, investigation of $\eta\eta$ channel in central $pp$ interaction by WA102 collaboration [28]; radiative $\phi$ decays at Novosibirsk [29], at Frascati by KLOE collaboration [30], and at the Thomas Jefferson National Accelerator Facility [31]; the $\eta'$ decay in $pp$ interactions at WASA/COSY [32]; the $J/\Psi$ decays by BES collaboration [33]; and charmless $B$ meson decays by Belle [34] and BABAR [35].
2. **Providing complementary support for lattice QCD:**
   Scalar mesons have been a topic of recent investigation in lattice QCD [2]. However, understanding the nature of scalar mesons which naturally involves various underlying mixing among quark and glueball states are too complicated for the current lattice calculations. My works on scalar mesons in the context of chiral Lagrangians provide an important role in guiding the lattice QCD investigations (please see Sec. 2).
3. **Providing broader impacts in elementary particle physics:**
   Physics of scalar mesons can provide significant insights outside its immediate focus of low-energy QCD, on important topics such as:
   - (a) Non-perturbative Higgs dynamics in Standard Model: As the dynamics of the light scalars in QCD is essentially described by the same Lagrangian of the Higgs boson in the minimal electroweak model, it is likely that the study of the scalar mesons sheds light on the mechanism of the electroweak symmetry breaking. Interesting connections are pointed out in [37].
   - (b) Role of scalar mesons in understanding the properties of heavier mesons, such as in recently investigated $D^*_{sJ}(2317)^+$ by BaBar Collaboration [38], and studies of the $D$ meson decays by the Fermilab experiment E791 [39].

In addition to the technical aspects discussed above, my research provides suitable theses/projects for graduate/undergraduate students. At SUNY Institute of Technology (which is primarily an undergraduate institute) I have involved undergraduate students in various computational aspects of my research (please see Subsection 2.4 and Sec. 4 for more details).

I have an ongoing research collaboration with Professor Joseph Schechter and his group at Syracuse University on different ways of probing the scalar mesons using chiral Lagrangians. The results of our investigations have been positively cited by many experts in the field, as well as the 2000, 2002, 2004, and 2006 editions of the Particle Data Group (PDG). To promote national and international research collaboration on this important topic in QCD, in May 2003, I organized an international workshop at SUNY Institute of Technology (SUNYIT) on this subject entitled "Scalar Mesons: An Interesting Puzzle for QCD," May 16-18, 2003, that included 23 presentations on scalar mesons by experts from around the world, and covered the most recent theoretical, experimental and computational developments on scalar mesons and related issues. (The proceedings is published by the American Institute of Physics

AIP Proc. **688**). Sponsoring such meetings will bring the latest advancements in the field to the host institute, and will lay out the foundation for a broad-based national and/or international collaboration. I have received continuous support from SUNY Institute of Technology to organize particle physics conferences/workshops. This has allowed me to organize seven particle physics gatherings hosted/sponsored by the SUNY Institute of Technology since September 2000 that I started my position at this institute:

- **The 27th Annual Montreal-Rochester-Syracuse-Toronto Meeting on High Energy Physics**
  May 16-18, 2005, SUNY Institute of Technology
  (http://mrst.sunyit.edu or http://www2.sunyit.edu/phy/05_mrst)
  Proceedings published by the World Scientific
  International Journal of Modern Physics A **20** (2005); ISSN: 0217-751X
  Editor: A.H. Fariborz
- **High Energy Physics Workshop "Scalar Mesons: An Interesting Puzzle for QCD"**
  May 16-18, 2003, SUNY Institute of Technology
  Partially supported by the U.S. Department of Energy (DE-FG02-03ER41263)
  (http://www.sunyit.edu/˜fariboa/03_SMW_HTML/index.html)
  Proceedings Published by the American Institute of Physics (2003)
  AIP Proc. **688**, ISBN 0-7354-0159-4
  Editor: A.H. Fariborz
- **The 25th Annual Montreal-Rochester-Syracuse-Toronto Meeting on High Energy Physics**
  May 13-15, 2003, Syracuse University
  (http://www.sunyit.edu/˜fariboa/03_MRST_HTML/index.html)
  Proceedings published by the American Institute of Physics (2003)
  AIP Proc. **687**, ISBN 0-7354-0161-6
  Editor: A.H. Fariborz
- **SUNY Institute of Technology Summer 2002 Conference on Theoretical High Energy Physics**
  Proceedings published by the NRC Research Press, National Research Council of Canada (2003)
  (ISBN 0-660-19065-6)
  Editors: M.R. Ahmady and A.H. Fariborz
- **Workshop on QCD Corrections in B Physics**
  SUNY Institute of Technology, June 2-23 (2002); Organizer: A.H. Fariborz
- **SUNY Institute of Technology Fall 2001 Conference on Theoretical High Energy Physics**
  Proceedings published by the SUNY Institute of Technology (2002)
  Editors: A.H. Fariborz and E. Rusjan
- **Ochanomizu University Workshop**
  Ochanomizu University, Tokyo, June 2001
  Proceedings published by the Mount Allison University (2002)
  Editors: M.R. Ahmady and A.H. Fariborz

These research and scientific activities have made SUNY Institute of Technology more visible regionally, nationally and internationally; have been endorsed by the American Physical Society; and for the first time SUNYIT was recognized as a high energy physics institute in the 2003-2004 list of "High Energy Physics Laboratories and Agencies," published by the Particle Data Group of the Lawrence Berkley National Laboratory (and continues to appear in this list since 2003).

My current research plans (that will be discussed in more details in Sec. 3) include investigating the existence and the quark substructure of the scalar mesons below 2 GeV and their impacts on related issues in low-energy QCD. Particularly, I am interested in relating different approximation and unitarization techniques, which in turn contribute toward our general understanding of different low-energy QCD frameworks. Also I would like to study the hadronic processes in search of exotic states, and the diquark interpretation of new resonances, as well as investigating the possible insights into the Higgs non-perturbative physics inspired by the strong dynamics of the scalar mesons in QCD.

# 2 Description of Previous Research

## 2.1 Main area of research:

Chiral Lagrangians, provide a powerful framework for studying the lowest and the next-to-lowest scalar states probed in different Goldstone boson interactions ($\pi\pi$, $\pi K$, $\pi\eta$,...) away from threshold. In this approach, a description of the scattering amplitudes which are, to a good approximation, both crossing symmetric and unitary is possible. To construct scattering amplitudes, all contributing intermediate resonances up to the energy of interest are considered, and only tree diagrams (motivated by large $N_c$ approximation) are taken into account. In this way, crossing symmetry is satisfied, but the constructed amplitudes should be regularized. Regularization procedure in turn unitarizes the scattering amplitude. By fitting the resulting scattering amplitude to experimental data, the unknown physical properties (mass, decay width, ...) of the light scalar mesons can be extracted. This large $N_c$ motivated approach provides a practical alternative to the chiral perturbation theory, and is applicable at energies away from threshold up to approximately 1.5 GeV.

There are 9 candidates for the lowest-lying scalar mesons ($m < 1$ GeV): $f_0(980)$ $[I = 0]$ and $a_0(980)$ $[I = 1]$ are the two which are well established experimentally [1]; $\sigma(550)$ or $f_0(600)$ $[I = 0]$ with uncertain mass and decay width [1]; and $\kappa(900)$ $[I = 1/2]$ which is not listed but mentioned in PDG [1]. The $\kappa(900)$ is observed in some theoretical models [41, 42, 43], as well as in some experimental investigations [24, 44]. It is known that a simple $q\bar{q}$ picture does not explain the properties of these mesons. Different theoretical models that go beyond a simple $q\bar{q}$ picture have been developed, including MIT bag model [45], $K\bar{K}$ molecule [46], unitarized quark model [42, 47], QCD sum-rules [48], and chiral Lagrangians [10, 26, 36, 41, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60]. The next-to-lowest scalar candidates (1 GeV $< m <$ 2 GeV) are: $K_0^*(1430)$ $[I = 1/2]$; $a_0(1450)$ $[I = 1]$; $f_0(1370)$, $f_0(1500)$, $f_0(1710)$ $[I = 0]$, and are all listed in [1]. The $f_0(1500)$ is believed to contain a large glue component and therefore a good candidate for the lowest scalar glueball state. These states, are generally believed to be closer to $q\bar{q}$ objects; however, some of their properties cannot be explained based on a pure $q\bar{q}$ structure.

In the context of the chiral Lagrangian of refs. [36, 41, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58], several low-energy processes that probe light scalar mesons are investigated. In order to describe the experimental data within this framework, there is a need for a $\sigma(550)$ and a $\kappa(900)$ in the analyses of $\pi\pi$ [49] and $\pi K$ [41] scattering, respectively. We have found that a large $N_c$ motivated treatment of $\pi K$ scattering can give a crossing symmetric and unitary amplitude as a fit to the existing experimental data. A novel feature of this approach, which is analogous to that employed for $\pi\pi$ scattering in [49], is to start with the invariant perturbative amplitude which is manifestly crossing symmetric. This results in individual contributions dramatically violating the partial wave unitarity bounds. Cancellations among these competing contributions rescues the unitarity. In our framework this suggests the existence of a light strange scalar resonance $\kappa$ which has a mass of 897 MeV and a width of 322 MeV. These results are similar to those of [43] in which a different model was employed. In addition, the fit for the $K_0^*(1430)$ properties also obtained is similar to that of the experimental analysis of [61]. By working to leading order in $1/N_c$, we can fit the real part of the partial wave amplitude to the experiment.

Motivated by the evidence for a $\sigma$ and a $\kappa$, and taking into account other experimentally well-established scalars – the $f_0(980)$ and the $a_0(980)$ – we have studied in [50] a possible classification of these scalars (all below 1 GeV) into a nonet $N$. By introducing a few new free parameters, we have studied the original effective Lagrangian in terms of $N$. The charge and iso-spin uniquely assign the physical states $I = 1$ and $I = 1/2$ to the off-diagonal elements of this nonet. However, the diagonal elements of $N$ give two isosinglet combinations $(N_1^1 + N_2^2)/\sqrt{2}$ and $N_3^3$, and therefore, the two physical iso-singlets $\sigma(550)$ and $f_0(980)$ are in general a linear combination of the diagonal elements which can be parametrized in terms of a scalar mixing angle $\theta_s$. A priori, the value of the mixing angle is not known and it is therefore a free parameter. In our framework, it is easy to see that the $\theta_s = \pi/2$ corresponds to an ideally mixed $q\bar{q}$ picture for $N$, whereas $\theta_s = 0$ corresponds to an ideally mixed $qq\bar{q}\bar{q}$ assignment (and can also allow a molecular structure). Therefore, depending on whether the best value for $\theta_s$ is closer to 0 or to $\pi/2$, we can probe the quark sub-structure of the nonet. We have shown that there exists a unique choice of the free parameters of this model which in addition to describing the $\pi\pi$ and $\pi K$ scattering amplitudes, well describes the experimental measurements for the $\eta' \rightarrow \eta\pi\pi$ decay [36]. The best value for the mixing angle consistent with the experimental data on these experiments is $\approx -20^o$, which is closer to an ideally mixed $qq\bar{q}\bar{q}$ assignment. The results of our investigations are in agreement with a number of current experimental and theoretical investigations. In particular, our 70 MeV estimate of the total decay width of $a_0(980)$ in [36] has been confirmed experimentally [62]. Also our prediction of a four quark nature for the lowest lying scalars is supported by the experimental and theoretical investigations of the radiative

$\phi$ decays [63, 64]. The need for a $\sigma(550)$ in our framework, is also supported by a recent experimental analysis [40] (also see [65]).

We have studied the $\pi\eta$ scattering within our model [51]. We have shown that this is a very suitable channel for learning more about the scalar mesons. We have examined the roughly elastic region (up to about 1.2 GeV) which is dominated by the $a_0(980)$ resonance. It is noteworthy that neither vector mesons nor large "current algebra" contact terms can contribute in this region, unlike the $\pi\pi$ and $\pi K$ cases. The non-trivial contributions all arise from light scalar meson exchanges. Thus $\pi\eta$ scattering seems an excellent channel for learning more about these resonances which are of great current interest. It is encouraging to us that treating the $a_0(980)$ in $\pi\eta$ scattering by the same method as used in earlier discussions of $\pi\pi$ scattering (in which a $\sigma(550)$ and the $f_0(980)$ appeared) and $\pi K$ scattering (in which a $\kappa(900)$ was needed) seems to be reasonable. We have also made a preliminary exploration of the nearby inelastic region (roughly $1-1.5$ GeV). This range features the $a_0(1450)$ scalar resonance. We have made similar preliminary discussions for the $\pi\eta \rightarrow K\bar{K}$ and $\pi\eta \rightarrow \pi\eta'$ off-diagonal processes. An additional complication shows up in the $\pi\eta \rightarrow K\bar{K}$ case. Here the vector meson $K^*$ exchange and a large current algebra contact term both contribute as in the $\pi\pi$ and $\pi K$ scatterings. Nevertheless we have found that the exactly crossing symmetric amplitudes for $\pi\eta \rightarrow \pi\eta$, $K\bar{K}$ and $\pi\eta'$ satisfied the unitarity relation amongst themselves to a reasonable accuracy until about 1.4 GeV.

We have extended our theoretical framework [52] to include a next possible scalar meson nonet which contains the $a_0(1450)$ and $K_0^*(1430)$ scalar mesons. We have studied the properties of these states (which are usually considered to belong to a conventional p-wave $q\bar{q}$ nonet in the quark model) in a framework where a lighter scalar nonet (of $qq\bar{q}\bar{q}$ type) was also present. We showed that certain puzzling features of these two particles could be naturally explained if the $q\bar{q}$ and $qq\bar{q}\bar{q}$ nonets mix with each other to form new physical states. The essential mechanism is driven simply by the fact that the isospinor is lighter than the isovector in the unmixed $qq\bar{q}\bar{q}$ multiplet. Although we carried out the analysis in a $qq\bar{q}\bar{q}$ picture for the unmixed light scalar nonet, it seems reasonable that it could also be done for other models of the light scalars (like the unitarized quark model [42, 47], or molecular models [46]) in which they have somewhat different four-quark interpretations. Indeed it seems likely that the mixing of $q\bar{q}$ states with $qq\bar{q}\bar{q}$ states already has a lot of similarity to the mixing with two meson states induced by unitarization in those schemes. For example the internal $qq\bar{q}\bar{q}$ wave function can be rewritten as a linear combination of color singlet $q\bar{q}\times$ color singlet $q\bar{q}$ and other pieces [50]. Moreover, this mixing provides a description of the mass spectrum and the partial decay widths of the $I = 1/2$ scalars [$\kappa(900)$ and $K_0^*(1430)$].

I have further extended this investigation to the case of $I = 0$ states in [66]. The properties of the $I = 0$ scalar states below 2 GeV [$\sigma(550)$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$] can be described within our chiral Lagrangian framework that is constrained by the properties of the $I = 1/2$ and $I = 1$ scalars, and in addition, includes terms that are relevant to $I = 0$ states only, such as mixing with a scalar glueball. The sub-structure of these states in terms of two and four quark components, as well as a glueball component is explored, and its correlation with the mass of $f_0(1370)$ is studied in [66]. Consistency with the available experimental data suggests that the $\sigma(550)$ is dominantly a non-strange four-quark state, whereas the sub-structure of other $I = 0$ states are sensitive to the input mass of $f_0(1370)$. This investigation estimates the scalar glueball mass in the range 1.47–1.64 GeV, consistent with the lattice QCD results [67]. I extended the theoretical framework of this investigation in ref. [68] in which the higher order mixing terms and their contributions to the mass matrix are calculated.

Most recently [69], I have extended the works of refs. [66, 68], and have investigated the correlations between the quark and glueball admixtures of the isosinglet scalar mesons below 2 GeV and the current large uncertainties on the masses of $\sigma(550)$ and $f_0(1370)$. Similar to the work of [66, 68], I have used a framework which is formulated in terms of two scalar meson nonets (a two-quark nonet and a four-quark nonet) together with a scalar glueball. I have shown that while some properties of these states are sensitive to the masses of $\sigma(550)$ and $f_0(1370)$, several relatively robust conclusions can be made: The $\sigma(550)$, the $f_0(980)$, and the $f_0(1370)$ are admixtures of two and four quark components, with $\sigma(550)$ being dominantly a non-strange four-quark state, and $f_0(980)$ and $f_0(1370)$ having a dominant two-quark component. Similarly, the $f_0(1500)$ and the $f_0(1710)$ have considerable two and four quark admixtures, but in addition have a large glueball component. For each state, I have given a detailed analysis providing the numerical estimates of all components. I have also shown that this framework clearly favors the experimental values: $m^{\rm exp.}[\sigma(550)] < 700$ MeV and $m^{\rm exp.}[f_0(1370)] = 1300-1450$ MeV. Moreover, with an overall fit to the available data I have shown that there is a reciprocal substructure for the $\sigma(550)$ and the $f_0(1370)$, and a linear correlation between their masses of the form $m[f_0(1370)] = 0.29\, m[\sigma(550)] + 1.22$ GeV. The scalar glueball mass of $1.5-1.7$ GeV is found in this analysis, consistent with the work of ref. [66] and the lattice QCD results [67]. The prediction of this work for the dominant four-quark and the sub-dominant two quark substructure of $\sigma(550)$ is consistent with the results of refs. [10, 26] in the context of unitarized chiral perturbation theory.

To learn more about the scalars, in collaboration with Professor Schechter and his group at Syracuse University, we have studied the role of the lowest-lying scalar nonet in the $\eta \rightarrow 3\pi$ decay process [70]. One might expect a large enhancement from diagrams including a light $\sigma(550)$. However there is an amusing cancellation mechanism which prevents this from occurring. In the simplest model there is an enhancement of about 13 percent in the $\eta \rightarrow 3\pi$ decay rate due to the scalars. In a more complicated model which includes derivative type symmetry breakers, the cancellation is modified and the scalars contribute about 30 percent of the total decay rate (although the total is not significantly changed). The vectors do not contribute much. Our model produces a reasonable estimate for the related $a_0(980) - f_0(980)$ mixing strength, which has been a topic of current debate.

We have studied the three flavor linear sigma model as a "toy model" for understanding the role of the light scalar mesons in the $\pi\pi$, $\pi K$ and $\pi\eta$ scattering channels [53]. The approach involves computing the tree level partial wave amplitude for each channel and unitarizing by a simple K-matrix prescription which does not introduce any new parameters. If the renormalizable version of the model is used there is only one free parameter. While this highly constrained version has the right general structure to explain $\pi\pi$ scattering, it is not quite right. A reasonable fit can be made if the renormalizability (for the *effective* Lagrangian) is relaxed while chiral symmetry is maintained. The occurrence of a Ramsauer Townsend mechanism for the $f_0(980)$ region naturally emerges. The effect of unitarization is very important and leads to "physical" masses for the scalar nonet all less than about 1 GeV. The $a_0(1450)$ and $K_0^*(1430)$ appear to be "outsiders" in this picture and to require additional fields.

In ref. [54], we have recently extended the investigation of ref. [53] to a general linear sigma model framework that includes two meson nonets (a two quark nonet and a four quark nonet). In this framework, we have studied a toy model in which corresponding chiral nonets (containing also the pseudoscalar partners) interact with each other. Although the "two-quark" and "four-quark" chiral fields transform identically under $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ transformations they transform differently under the $\mathrm{U}(1)_A$ transformation which essentially counts total (quark + antiquark) content of the mesons. To implement this we have formulated an effective Lagrangian which mocks up the $\mathrm{U}(1)_A$ behavior of the underlying QCD, and have derived generating equations which yield Ward identity type relations based only on the assumed symmetry structure. We have applied this to the mass spectrum of the low lying pseudoscalars and scalars as well as their "excitations". Assuming isotopic spin invariance, it is possible to disentangle the amount of"two-quark" vs."four-quark" content in the pseudoscalar $\pi, K, \eta$ type states and in the scalar $\kappa$ type states. We have found that a small "four-quark" content in the lightest pseudoscalars is consistent with a large "four-quark" content in the lightest of the scalar $\kappa$ mesons. This toy model also allows one to easily estimate the strength of a "four-quark" vacuum condensate. We have further extended this investigation in ref. [55], in which a detailed estimate of the mass spectra of the $I = 0$, $I = 1/2$ and $I = 1$ pseudoscalars, and the $I = 1/2$ scalars are studied, and numerical estimates of their quark content are presented. We have found, as expected, that the ordinary and the excited pseudoscalars generally have much less two and four quark admixtures compared to the respective scalars. As by-products, we have given several numerical estimates for quantities such as the four-quark vacuum condensate, and the decay constant of excited pion, kaon and kappa states.

We have further extended these investigations [56] by introducing a systematic procedure for selecting terms from the general chiral Lagrangian of ref. [54]. We applied this approach, which is based on modeling of the $U(1)_A$ and suppressing the effective vertices with large number of quark lines, to study scalar and pseudoscalar meson masses and quark substructure in [57] in the limit of zero quark masses. We found that the scalar mesons contain a large four-quark content whereas the pseudoscalars are dominantly two-quark states. We then investigated this model when a SU(3) symmetric quark mass is turned on [58]. We noted that the current algebra formula no longer holds exactly. We also calculated the s wave scattering lengths, including beyond current algebra theorem corrections due to the scalar mesons, and observed that the model can fit the data well. This study uncovered the way in which the linear sigma models give controlled corrections (due to the scalar mesons) to the current algebra scattering formula, providing an alternative to the nonlinear sigma model approach.

We have studied the K-matrix unitarization of the SU(2) linear sigma model [37], which can explain the experimental data in the scalar $\pi\pi$ scattering channel of QCD up to about 800 MeV. Since it is just a scaled version of the minimal electroweak Higgs sector, which is often treated with the same unitarization method, we interpret the result as support for this approach in the electroweak model with scaled values of tree level Higgs mass up to at least about 2 TeV. The relevant QCD effective Lagrangian which fits the data to still higher energies using the same method involves another scalar resonance. This suggests that the method should also be applicable to corresponding beyond minimal electroweak models. Nevertheless, even with one resonance, the minimal K matrix unitarized model behaves smoothly for large bare Higgs mass by effectively integrating out the Higgs while preserving unitarity.

## 2.2 Other related research:

The chiral Lagrangian approach presented in Subsection 2.1, probes scalar mesons in different pseudoscalar-pseudoscalar scatterings and decays. We saw that even though the chiral Lagrangians are formulated in terms of the meson fields and in principle do not know anything about the underlying quark substructure, the knowledge of the scalar mixing angle provides a probe into the quark substructure of these states. To support/test this probe, it is important to investigate the quark combination of these states based on a theory that is formulated at the quark level. For this purpose the QCD sum-rules are powerful candidates. A significant part of my Ph.D. thesis was devoted to the application of the QCD sum-rules to the scalar channel in order to see if the QCD sum-rules are consistent with the existence of a broad and light $\sigma$ resonance [71, 72]. I also worked on the application of QCD sum-rules to study the properties of the lowest pion resonance [73]. There are certainly important links between these two lines of research: the chiral Lagrangians (which are effective theories and provide a convenient way of describing data), and the QCD sum-rules (which are formulated at the quark level and probe the QCD vacuum).

In collaboration with late Professor V. Elias of the University of Western Ontario and Professor T.G. Steele of the University of Saskatchewan, we examined the scalar channel for the properties of the lowest-lying scalar mesons, as well as the pseudoscalar channel for the properties of the radial excitation of the pion. In [71], we presented the sum rule methodology necessary to address the physical properties of the lowest-lying scalar mesons. We showed how nonzero resonance widths can be incorporated into the hadronic contribution to QCD Laplace sum rules, which are particularly appropriate for studying lowest-lying resonance properties, and demonstrated explicitly how a lowest-lying resonance's nonzero width elevates a sum rule determination of that resonance's mass. We presented the field-theoretical content of appropriate scalar current sum rules, and discussed the sum rule contribution arising from the 3-loop order purely-perturbative QCD contributions to the scalar current correlation function. We presented nonperturbative sum rule contributions arising from QCD-vacuum condensates and direct single-instanton contributions to the $I = 0$ and $I = 1$ scalar current correlation functions. We utilized these results to obtain a sum-rule determination of the masses of lowest-lying scalar resonances. Stability curves were generated leading to estimates of such masses for a given choice of width and the continuum threshold above which perturbative QCD and hadronic physics are assumed to coincide. Detailed comparison was made with earlier sum-rule generated stability curves [74], showing how the separate incorporation of renormalization-group improvement, 3-loop perturbative effects, nonzero widths, and the contribution of instantons individually affect such curves. We examined the isoscalar channel in further detail by obtaining values of the mass, width, continuum threshold, and coupling of the lowest-lying $q\bar{q}$ resonance via a weighted least-squares fit to the overall Borel-parameter dependence of the first Laplace sum rule. A relationship between the anticipated resonance coupling and a phenomenologically estimable matrix element was developed and was utilized to obtain an estimate of the light quark mass. We presented our conclusions concerning the questions on the nature of the lowest-lying scalars in [71]. We also assessed the compatibility of sum rule predictions for the lowest-lying non-exotic $I = 1$ resonance with $\mathrm{a}_0(980)$ and $\mathrm{a}_0(1450)$. We concluded that the possibility of having a broad and light $q\bar{q}$ sigma, as well as a $q\bar{q}$ structure for $a_0(980)$ are ruled out, and that the $a_0(1450)$ has a substantial quark-antiquark component. These conclusions are clearly consistent with the results obtained in the chiral Lagrangian approach described in Sec. 2.1, in which the lowest-lying scalar states are not simple $q\bar{q}$ objects.

In [72], we further explored the scalar mesons in the context of the QCD sum-rules. We performed a Hölder inequality analysis of the QCD Laplace sum-rule which probes the non-strange ($n\bar{n}$) components of the $I$=0,1 (light-quark) scalar mesons, and showed that this inequality supports the methodological consistency of an effective continuum contribution from instanton effects. This revised formulation enhances the magnitude of the instanton contributions which split the degeneracy between the $I = 0$ and $I = 1$ channels. Despite this enhanced isospin splitting effect, analysis of the Laplace and finite-energy sum-rules seems to preclude identification of $a_0(980)$ as a state with predominant $n\bar{n}$ components. In this work, the modeling of the hadronic part of the correlation function, an important component of the QCD sum-rule analysis of a broad resonance, was inspired by the nonlinear chiral Lagrangian framework of refs. [41, 49, 50, 36, 51, 52, 53]. This is an example of important links between the chiral Lagrangians and the QCD sum-rules, and is recently supported by an investigation by Achasov [75].

I have also worked on the study of the first pion excitation, or $\Pi(1300)$ resonance, in the context of the QCD sum-rules. In [73] we studied this resonance as a sub-continuum resonance in the pseudoscalar channel, and obtained parameters characterizing this resonance through a global fit of the Borel-parameter dependence of the field-theoretical pseudoscalar Laplace sum rule to its hadronic (pion + pion-excitation + QCD-continuum) content. Our analysis incorporated finite-width deviations from the narrow resonance approximation, instanton effects, and higher-loop perturbative contributions to the pseudoscalar correlator. We obtained the following values (uncertainties reflect 90% confidence levels): mass $M_\Pi = 1.15 \pm 0.28$ GeV, width $\Gamma_\Pi =< 0.48$ GeV, decay constant $r \equiv [F_\Pi M_\Pi^2 / f_\pi m_\pi^2]^2 = 4.7 \pm 2.8$. This small value of decay constant can be understood based on a mixing mechanism between the lowest and the next-to-lowest lying scalar meson nonets in the context of the linear sigma model in [53], in which the quark content of the $\Pi(1300)$ is estimated. This is another example of important links between the chiral Lagrangians and the QCD sum-rules.

Finally, I have worked on non-perturbative dynamics of light quarks that can provide useful insights into the QCD dynamics at low energies. Based upon Higgs boson low-energy theorems, the coupling of the light Higgs to nucleons has been estimated in the literature [76]. In a collaborative work in [77], we investigated the possibility of estimating the same coupling based upon a perturbative QCD approach. Within this framework, all non-perturbative effects are incorporated into a dynamical mass for the light quark inside the nucleon. We found that the low-energy theorem results can be recovered provided that we use a realistic form for the behavior of the dynamical mass of the light quark. This model also sheds light on the magnitude of the strong coupling at low energies. This approach can be extended to the coupling of a massive Higgs boson to light quarks, an issue that Higgs low-energy theorems fail to address.

### 2.3 Other research:

I have had other research activities in particle physics that may not be directly related to my main research interest of low-energy QCD. These include, Pade approximant techniques in estimating the three-loop contributions to the inclusive semileptonic $b \to c$ decay rate [78], perturbative corrections in $b \to u$ semileptonic decays [79], study of logarithmic contributions to semileptonic B-decays [80], renormalization group improvement of the perturbative series for the $e^+ e^-$ annihilation cross-section [81], neutrino physics and understanding the recent experimental data on neutrino oscillation [82]; dileptonic decay of B meson to various resonances of the K meson [83]; and issues pertinent to our understanding of fermion fields on a gravitational background [84].

### 2.4 Involving undergraduate students in research:

My research in general involves a significant amount of numerical and symbolic computation. For numerical work and data analysis, I have mainly used the FORTRAN language, and have extensively worked with many numerical packages and libraries including IMSL, Numerical Recipes, and MINUIT. For symbolic work, I have used the MAPLE package and have written numerous codes that perform certain computations in particle physics, such as form factor computation and Feynman integrals [85]. The computational aspects of my research have provided suitable projects for students at SUNYIT, including algorithm design for various symbolic computations in the framework of linear and nonlinear chiral Lagrangians; algorithm design for multi-parameter minimization; and parallel computation. I have explored the local resources at SUNYIT and have obtained funding to support undergraduate students. Since September 2000, I have supervised 13 students (Melissa Commisso, Alex Diperna, Patricia Dozois, George W. Hippisley, Paul Lein, Thomas Lenahan, Rachel Marilley, Michael McDonald, Christopher McGrath, Aaron Reimann, Gennady Staskevich, Vishal Shah and Rosemary Siebold).

I plan to continue working with undergraduate students to develop several numerical and symbolic codes to perform the chiral Lagrangian computations regarding scalar mesons. Please see Sec. 4 for more details on specific undergraduate research projects. (More details on student projects are available at http://www.sunyit.edu/~fariboa/students ).

## 3 Research Plans

In future, I will try to take advantage of any new opportunities in which I can apply my experience of low-energy QCD. In addition, I would be interested in devoting at least some portion of my time to explore the existence and the quark substructure of the light scalar mesons and their impacts on related issues in low-energy QCD. Particularly, relating different approximation and unitarization techniques is an important step in this direction, and it also contributes toward our general understanding of different low-energy QCD frameworks. I would like to explore hadronic processes in search of exotic states, and investigate the diquark interpretation of new resonances. I would also like to investigate possible connections between the physics of scalar mesons and non-perturbative Higgs dynamics in electroweak theory. More specific descriptions are given below.

### 3.1 Exploring the properties of the $I = 0$ scalar mesons and the scalar glueball:

Lattice QCD [67], as well as chiral Lagrangian approach [66, 69], estimate the scalar glueball mass in the range 1.4 - 1.7 GeV. In this range of energy, there are several isosinglet states [$f_0(1370)$, $f_0(1500)$, $f_0(1710)$] that strongly mix with the scalar glueball, as well as with the lowest-lying isosinglet scalar states $\sigma(550)$ and $f_0(980)$. Therefore, in order to understand the properties of scalar glueball, it is necessary to develop a framework in which all such mixings are taken into account. This is, both technically as well as computationally, a challenging problem and will provide valuable information about the non-perturbative QCD dynamics. It will provide a parallel approach to lattice QCD on several issues such as reducing the uncertainties on the glueball mass, estimating the mixing strength between the glueball and the $\bar{q}q$ scalars (see [86] for a lattice estimate), and estimating the glueball mixing strength with the $\bar{q}\bar{q}qq$ scalars. For this purpose, I have done a preliminary investigation in refs. [66, 69], using the chiral Lagrangian of refs. [41, 49, 50, 36, 51, 52]. These works have already provided promising results, and more investigation on the scalar glueball can be done in this framework. To further reduce the theoretical uncertainties, several higher order effects should be included. For example, inclusion of the higher order effective mixing terms between the lowest four-quark nonet $N$ and the next-to-lowest two-quark nonet $N'$ is an important issue to study. In the two nonet treatments of $I = 1/2$ and $I = 1$ scalar mesons in ref. [52], the leading mixing term between the two and four quark nonets is used and fairly good agreement with experiment is obtained. The next-to-lowest mixing terms in the effective Lagrangian include terms such as $\mathrm{Tr}\left[\mathcal{M}\left(NN' + N'N\right)\right], \mathrm{Tr}\left(\mathcal{M}N\right)\mathrm{Tr}\left(N'\right), \mathrm{Tr}\left(\mathcal{M}N'\right)\mathrm{Tr}\left(N\right), G\mathrm{Tr}\left(\mathcal{M}N\right)$, and $G\mathrm{Tr}\left(\mathcal{M}N'\right)$, where $\mathcal{M}$ is the SU(3) symmetry breaker. Also using the framework of refs. [66, 69] and incorporating new experimental data on several two-pseudoscalar decay channels of isosinglet scalars below 2 GeV, the glueball content of these states, as well as their two and four quark components, can be estimated.

Specifically, I would like to extend the prior works on the two scalar meson nonet mixing of refs. [52, 66, 69]. In ref. [68], the general form of the nonlinear chiral Lagrangian for the lowest lying and the next-to-lowest lying scalars are given, and the mass matrices in the general case in which all SU(3) symmetry breakers are taken into account, are presented. Using the results of ref. [68], I would like to generalize prior works [52, 66, 69] wherein the higher order symmetry breakers were not considered.

The mass matrices for the isodoublet and isotriplet states [$\kappa(900), K_0^*(1430), a_0(980)$ and $a_0(1450)$] depend on six unknown parameters, and there are four inputs for the physical masses. The general mass Lagrangian for the $I = 0$ states below 2 GeV [$\sigma(550)$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$] in terms of the two and the four quark nonets, and a scalar glueball, contains 12 a priori unknown parameters, in addition to the six unknown parameters that appear in the mass Lagrangian for the isodoublet and isotriplet states and contribute to the isosinglet masses as well. (Out of the 12 parameters, seven describe the mixing between the two and the four quark nonets, and the mixing of these two nonets with a scalar glueball [68].)

It would be interesting to extend this investigation to include all higher order effects that were ignored in the first approximation. In the general case, there are altogether 28 free parameters, that can be fixed by a number of experimental inputs (or theoretical bounds), such as the 9 scalar masses ($m[\kappa(900)]$, $m[K_0^*(1430)]$, $m[a_0(980)]$, $m[a_0(1450)]$, $m[\sigma(550)]$, $m[f_0(980)]$, $m[f_0(1370)]$, $m[f_0(1500)]$ and $m[f_0(1710)]$, several decay ratios and decay widths (such as the two pion decay of $f_0(980)$, the $\pi\eta$ decay of $a_0(980)$, $\eta' \to \eta 2\pi$ decay, $\cdots$ together with the decay ratios given, for example, by WA102 collaboration [28]), as well as a number of pseudoscalar scattering amplitudes (such as $\pi\pi$ and $\pi K$ scattering amplitudes). It is one of my objectives to compute these parameters, and consequently determine with relatively good accuracy the quark and glueball contents of the scalar mesons. Probing scalar-pseudoscalar-pseudoscalar couplings is important in a number of low energy processes such as $\eta \to 3\pi$ and $\eta' \to 3\pi$, in which the scalar mesons are expected to play important roles [36, 70]. This project will involve an extensive computation and requires developing several numerical as well as computer algebra codes. For the earlier stages of this project [66, 69], I have developed numerical algorithms in FORTRAN codes, have written numerical codes that access the IMSL library for parameter search, and have developed computer algebra codes in MAPLE for this purpose. In addition, I have worked with students in applied mathematics and computer science at SUNYIT to parallelize some of these codes, and have obtained a promising initial test-run results.

### 3.2 Revisiting the $\eta \to 3\pi$ and $\eta' \to 3\pi$ Decays:

The study of $\eta \to 3\pi$ is known to be surprisingly complicated and correspondingly important for understanding the non-perturbative (low energy) structure of QCD. There is a significant difference between theory and experiment for the $\eta \to \pi^0\pi^+\pi^-$ width: The experimental width [1] is $\Gamma_{0+-} = 267 \pm 25$ eV, while the one loop chiral perturbation theory result [87] suggests $160 \pm 50$ eV. Other attempts [88] to estimate final state interaction effect outside of the chiral perturbation theory approach have increased this somewhat to $209 \pm 20$ eV.

It is of great interest to see what roles scalars play in this process. In the first attempt of applying our framework for chiral Lagrangian of scalar mesons, we have studied the role of possible light scalars in the $\eta \to 3\pi$ decay in [70]. Typically this process has been treated by chiral perturbation theory [87], in which the possible effects of scalars have been amalgamated into effective contact interactions among the pseudoscalars. To learn about the scalars it is natural to keep them rather than integrating them out [89, 70]. Number of important issues should be investigated for this decay. The possibility that a light scalar [like the $\sigma(550)$] might give an enhancement due to closeness of its propagator to the pole is an issue that requires a careful treatment. It is also important to study the $\eta \to 3\pi$ decay, in order to become more familiar with the isospin violating $a_0(980) - f_0(980)$ transition which plays a role in this decay. Still another reason for the interest in the effects of the scalars in $\eta \to 3\pi$ decay is to provide a connection with the discussion of the apparently puzzling $\eta' \to 3\pi$ decays in which light scalar mesons can be reasonably expected to have very large effects. I plan to investigate these issues. Particularly, other components of my research, such as the mixing between the lowest and the next-to-lowest scalar meson nonets in the nonlinear chiral Lagrangian framework; the coupled channel analysis of pesudoscalar interactions; and the two-nonet formulation of the linear sigma model will provide important insights into the $\eta \to 3\pi$ decay.

A particularly interesting scalar contribution to $\eta \to 3\pi$ arises from the $a_0 - \sigma$ transition (The $a_0 - f_0$ transition contribution to $\eta \to 3\pi$ is suppressed due to the propagator of the heavier $f_0(980)$). This is the analog of the important $\pi^0 - \eta$ transition and, in a sense, is a new mechanism for $\eta \to 3\pi$ (although it was investigated a long time ago [90] as a possible way to increase the $\eta \to 3\pi$ width). This method also evaluates the strength of the $a_0(980) - f_0(980)$ transition. This transition has been suggested to be relevant in study of $\Gamma(\phi \to f_0\gamma)/\Gamma(\phi \to a_0\gamma)$ ratio [91, 92]. Intuitively, because of the near degeneracy of the $a_0(980)$ and $f_0(980)$ as well as the similarity of their widths, one might expect the mixing to be very large. But the mixing amplitude is suppressed by the scalar meson width. I plan to study the effect of the final state interaction of the pseudoscalars, and that can provide useful information on the effect of the widths of the $a_0(980)$ and $f_0(980)$, and their possible role in the $\eta \to 3\pi$ decay.

From the standpoint of learning more about the properties of the scalar mesons it is clear that the $\eta' \to 3\pi$ decays represent a potentially important source of information. In this case there is sufficient energy available for the $a_0(980)$ and $f_0(980)$ propagators to be close enough to their mass shells to avoid suppressing the contributions of these resonances. On the other hand, the theoretical analysis is more difficult since large non-perturbative unitarity corrections are expected. In addition, other more massive particles may also contribute. The experimental information is more preliminary than in the $\eta \to 3\pi$ case. While a number with reasonably small errors has been presented for $\Gamma'_{000}$, there is only a weak upper bound for $\Gamma'_{+-0}$ and also no information on its Dalitz plot. In our model the $\eta' \to 3\pi$ amplitudes are related to the $\eta \to 3\pi$ amplitudes. I plan to study the properties of the heavier scalar states [such as the $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$] and their mixing with the lowest-lying scalars, and this in turn can shed some light on the $\eta' \to 3\pi$ decay and can determine whether any significant contribution to this decay may come from the heavier scalars and their mixing with the light scalars.

### 3.3 Two nonet formulation of the linear sigma model:

In ref. [53], a possible description of the mass and quark substructure of the scalar mesons is given in the context of the linear sigma model formulated in terms of two meson nonets. This formulation also gives a probe into the quark substructure of both the light as well as the heavy $I = 1$ scalar and pseudoscalar states. This work was further extended in [54, 56, 57, 58] in which a general approach based on the underlying symmetries and the mechanisms of their breakdown is presented.

With my collaborators, we plan to investigate several important issues in this framework, including the quark substructure of the light and the heavy pseudoscalar and scalar mesons below 2 GeV, non-perturbative QCD vacuum, scalar mixing (including the possibility of mixing with glueballs for the $I = 0$ states), and four quark condensates (an issue of great interest in the QCD sum-rules). It seems interesting to apply this two nonet framework to the problem of the pseudoscalar scatterings, and since the unitarization procedure played an important role in our analysis it seems very desirable to investigate more "dynamical" methods than the conventional K-matrix scheme employed in [53]. The most general chiral Lagrangian will be considered and independent of any specific assumption for the form of the Lagrangian, the above issues will be investigated. This will require developing computational techniques that incorporate the underlying symmetries to extract general results. One of the important points about this project is the fact that this model treats the lowest and the next-to-lowest nonets of both the scalar as well as the pseudoscalar mesons, and therefore will be constrained by a number of available experimental data. The general treatment of the scattering of pseudoscalar mesons that include a general mixing mechanism between the lowest and the next-to-lowest states can provide, in the low energy limit, an estimate of the corrections to chiral perturbation theory results. Also as this model probes the diquark dynamics, it can provide, in a limiting approach, a connection with the QCD at nonzero densities and temperatures.

In collaboration with Professor J. Schechter, we hope to extend the works of refs. [54, 56, 57, 58]. Clearly, there are a number of interesting directions for further work. We plan to add SU(3) symmetry breaking mass terms in the same systematic scheme employed in these references. Mixing with glueball states and possibly other chiral nonets is also an intriguing possibility. Of course, an important ingredient to be taken into account would be the changes in the model parameters which result from unitarizing the tree level scattering amplitudes and comparing with the unitarized amplitudes with experiment. This was carried out for the 2 flavor Gell Mann - Levy model in [37] and for the 3 flavor single M model in [53].

### 3.4 Possible insights into the nonperturbative Higgs dynamics inspired by the physics of scalar mesons:

The K-matrix unitarized linear SU(2) sigma model explains the experimental data in the scalar channel of QCD up to about 800 MeV [53]. Since it is just a scaled version of the minimal electroweak Higgs sector, which is often treated with the same unitarization method, we have investigated in [37] the possibility that there is a support for this approach in the electroweak model up to at least Higgs bare mass about 2 TeV. The relevant QCD effective Lagrangian needed to go higher in energy is more complicated than the SU(2) linear sigma model and is better approximated by the linear SU(3) sigma model [53]. This enabled us in [53] to extend the energy range of experimental agreement at the QCD level by including another scalar resonance. The better agreement at larger energies in the QCD model could give support to a similar treatment for a perhaps more realistic Higgs sector in the electroweak theory which may be valid at higher energies due to additional higher mass resonances.

Motivated by the encouraging results that we have obtained on the dynamics of scalar mesons within chiral Lagrangian frameworks (described in Sec. 2), as well as the interesting connections with the Higgs physics that we have pointed out in ref. [37], it would be worthwhile to further investigate such connections and providing more insights into the nonperturbative Higgs dynamics. Particularly, it would be interesting to investigate possible connections between the two-nonet linear sigma model of the scalar mesons in QCD (described above), and the electroweak models with more than one Higgs particle. There are many candidates in the electroweak theory, such as, for example, larger Higgs sectors, larger gauge groups, supersymmetry, grand unified theories, technicolor, string models, and symmetry breaking by background chemical potentials which may give rise to more than one Higgs particle in the same channel.

### 3.5 QCD sum-rules probe of the $\kappa(900)$ meson:

$\kappa(900)$ scalar meson is seen in some theoretical models, including in one of my collaborative works in the context of a nonlinear chiral Lagrangian study of $\pi K$ scattering in [41]. This scalar is a broad resonance and although weakly couples to the $\pi K$ channel, it is a crucial ingredient in fitting the nonlinear chiral Lagrangian prediction of the $\pi K$ scattering amplitude to experiment. In particular its interference with the $K_0^*(1430)$ is essential in understanding the available data. It is therefore, a very interesting (and non-trivial) problem to probe the quark substructure of the $\kappa(900)$ in the context of the QCD sum-rules. The important challenge in QCD sum-rule study of this resonance is the incorporation of the broad resonance width, and its interference with the $K_0^*(1430)$. I have worked on this resonance in the context of chiral Lagrangians, and I would also like to study its properties within the QCD sum-rules framework. I have previously worked on QCD sum-rule study of $\sigma(550)$ in [71, 72] and I believe similar investigations can be carried out for the $\kappa(900)$. A preliminary investigation is given in [93].

# 4 Research Plans Involving Students

## 4.1 Introduction:

Computations in linear and nonlinear chiral Lagrangians are often complicated and time consuming due to nonlinearities of the Lagrangian as well as the combinatorics. Therefore, it is important to develop efficient algorithms that can perform such computations. Over the past eight years, I have worked with a number of undergraduate students at SUNYIT on algorithm design for various problems that I have been working on in particle physics. In these projects, students gain research experience and are able to significantly contribute to various stages of the work. In addition to physics students, students in applied mathematics, computer science and engineering programs can also contribute to these projects. It is very important to note that these types of computational projects are very suitable for undergraduate students because the goal of an algorithm design is to come up with a way of reducing a complex calculation into a sequence of logical steps which are usually quite stimulating and are simply understood by undergraduate students. We know that calculating a physical quantity in quantum field theory has different levels of computation which usually starts out from some Feynman diagrams, but eventually reduces down to ordinary algebra and calculus (even though often quite messy and tedious). Therefore, although undergraduate students do not usually know quantum field theory, the quantum field theory problems can always be reduced to a mathematical level which is accessible to undergraduate students. A natural way to overcome a messy algebra and a tedious calculus is to employ numerical and/or symbolic computational techniques. Therefore, it is a rewarding activity to invite undergraduate students to participate in such projects, as the objectives are easily understandable, the mathematical/computational methods are accessible and stimulating, and therefore students enjoy working on these projects and making a significant contribution to the field.

Over the years, I have developed algorithms for several specific computations in chiral Lagrangian frameworks. Some of these works are at various stages of completion and undergraduate students can significantly assist me to achieve these objectives. In subsections 4.2 and 4.3, I have given several specific projects that will involve undergraduate students.

## 4.2 Algorithm design for calculation of S-matrix elements for various interactions in nonlinear chiral Lagrangians:

Nonlinear chiral Lagrangians are formulated in terms of the nonlinear field $U = e^{\frac{2i\phi}{F_\pi}}$ with $\phi$ being the pseudoscalar meson nonet and $F_\pi$ the pion decay constant. To calculate the S-matrix for a given process one can divide the computation into three main steps:

1. Express the Lagrangian in terms of matrix $\phi$ and collect the appropriate terms
2. Extract the operators that contribute to a particular process
3. For each operator evaluate the S-matrix

Taking into account the fact that the Lagrangian has several sectors (ex. pseudoscalar sector, vector sector, scalar sector, ...) and each sector usually consist of a number of terms (each of which a function of $U$), it is easy to see that for a given physical precess there is an avalanche of contributions to S-matrix. Therefore, it is highly advantageous to develop computer algebra codes for each of the three steps of S-matrix calculation given above. Below I have given two examples to demonstrate the importance of computer algebra techniques in S-matrix calculation in nonlinear chiral Lagrangian frameworks (some of these codes are available at: http://www.sunyit.edu/˜fariboa/students ).

### 4.2.1 Example 1: S-matrix element for $K^+ K^- \to K^+ K^-$ from the leading kinetic term:

The leading kinetic term for the pseudoscalar Lagrangian is

$$\mathcal{L} = \frac{F_\pi^2}{8}\,\mathrm{Tr}\,\left(\partial_\mu U\,\partial_\mu U^\dagger\right) + \cdots \tag{1}$$

where the dots represent higher derivate terms. (In addition, the pseudoscalar sector contains terms which are powers of $U$ and $U^\dagger$.) Implementation of the above three steps are:

1. $\mathcal{L} = \frac{1}{3F_\pi^2}\mathrm{Tr}\,\left((\partial\phi)(\partial\phi)\phi^2 - (\partial\phi)\phi(\partial\phi)\phi\right) + \cdots$
2. The above $\phi^4$ part of the Lagrangian, contains $2 \times 3^4 = 162$ operators, but not all contribute to the process $K^+K^- \to K^+K^-$. We can show that the contributing operators are:

   $$\mathcal{L} = \frac{1}{3F_\pi^2}\left(2\partial_\mu K^+\partial_\mu K^- K^+ K^- - \partial_\mu K^+\partial_\mu K^+ K^- K^- - \partial_\mu K^-\partial_\mu K^- K^+ K^+\right) + \cdots$$

3. Having identified the relevant operators, we evaluate the S-matrix for each operator:

   $$\langle K^+(p_1')K^-(p_2')|i\mathcal{L}|K^+(p_1)K^-(p_2)\rangle$$

   in which all possible combinatorics should be taken into account, and that is usually rather time consuming and prone to error.

Each of the three steps can be accurately computed using a computer algebra algorithm. For example, following quantum field theory, we can show that step 3 effectively comes down to taking all possible partial derivatives on the incoming and outgoing waves. A program that can achieve this goal can be developed, and I have done an initial study and have designed an underlying algorithm which performs this computation. This can be implemented on a computer algebra package such as MAPLE (the algorithm is available at http://www.sunyit.edu/˜fariboa/students ).

With student assistant I am hoping to be able to extend this algorithm, and also develop algorithms for step 1 and step 2 of the S-matrix calculation. This will result in a series of effective codes that eventually can lead to creating a library/package for such S-matrix applications.

### 4.2.2 Example 2: S-matrix element for $\eta \to \pi^0\pi^+\pi^-$ process from vector contribution:

As a second example, we consider the S-matrix element for the $\eta \to \pi^0\pi^+\pi^-$ process. Among numerous other contributions, vector mesons make several contributions to this S-matrix, including the contribution from a term in vector Lagrangian proportional to [70]:

$$\mathrm{Tr}\,\left(\hat{M}_+\,p_\mu\,p_\mu\right) \tag{2}$$

where $p_\mu = \frac{i}{2}\left(\xi\,\partial_\mu\,\xi^\dagger - \xi^\dagger\,\partial_\mu\,\xi\right)$, $\hat{M}_+ = \frac{1}{2}\left(\xi\,\mathcal{M}\,\xi + \xi^\dagger\,\mathcal{M}\,\xi^\dagger\right)$, $\mathcal{M}$ is a diagonal matrix representing the quark masses, and $\xi^2 = U$. The three stages of S-matrix computation for this particular case are:

1. The relevant operators in terms of $\phi$ matrices are:

   $$\mathrm{Tr}\,\left(\mathcal{M}\ \partial_\mu\phi\ \partial_\mu\phi\ \phi^2\right) + \text{five permutations of partial derivatives} + \cdots$$

2. The total number of operators in the above $\phi^4$ terms are: $6 \times 3^4 = 486$, out of which, $6 \times 12 = 72$ operators contribute to the $\eta \to \pi^0\pi^+\pi^-$ process. In addition, due to the $\eta - \eta'$ mixing, as well as the effective isospin-mixing vertices $\pi^0 \cdot \eta$ and $\pi^0 \cdot \eta'$, effectively, $6 \times 12 \times 8 = 576$ physical operators contribute to this decay which originate from Eq. (2).
3. For each of the above operators the S-matrix should be calculated. (This process also involves keeping track of permutation factors for the matrix elements.)

Again, in this example, we see that a reliable algorithm can be very useful when analyzing an important problem that involves a long and tedious algebra. To revisit $\eta \to 3\pi$ decay, undergraduate students can assist me to continue to develop effective algorithms for this project. (More details available at http://www.sunyit.edu/˜fariboa/students )

### 4.3 Algorithm design for linear sigma model computations:

Linear sigma model provides a powerful framework for studying the properties of the scalar and pseudoscalar mesons. However, similar to the nonlinear chiral Lagrangians, the computations can be very lengthy and therefore it is very efficient to develop a set of strong algorithms for carrying out the repetitive computations. To achieve this goal, undergraduate students in physics, mathematics, and engineering can significantly contribute to various stages of the algorithm development. I have designed algorithms for computing various scalar-pseudoscalar-pseudoscalar and four-pseudoscalar coupling constants which are essential ingredients in computing different scatterings or decays. The main complication is due to combinatorics involved in the generating equations that relate, for example, the $\phi^4$ coupling to 31 functions of the scalar-pseudoscalar-pseudoscalar couplings and meson masses. I would like to continue extending these algorithms to a library for the linear sigma model. Undergraduate students can significantly help me to achieve this goal. (Several algorithms for linear sigma model computations are given at: http://www.sunyit.edu/~fariboa/students )

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