# Applied Mathematics Seminar

**Friday, April 15, 2pm, DON 2159**

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## A Geometric Theory of Nonlinear Morphoelastic Shells

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In this work, a geometric theory of nonlinear morphoelastic shells is formulated to investigate the effects of bulk growth and remodelling on the evolution of the geometry and the induced residual stresses in two dimensional nonlinear elastic bodies.
Growth and remodelling are particularly important in biological tissues. Indeed, morphoelasticity and its coupling with the induced deformations and mechanical stresses are crucial for the working conditions and the physiological functions of living organs.
A general theory is proposed to account for both membranar and bending effects through the intrinsic and extrinsic geometries of the shell.

This talk is based on joint work with Arzhang Angoshtari (GWU), Alain Goriely (Oxford), and Arash Yavari (Georgia Tech).

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Students are very welcome to the informal discussion seminar. We assume knowledge in mathematics and physics at the level of Calculus III and Physics II.

## More information about the talk

In fact, the shell is taken as an embedded hypersurface of the three-dimensional thin body of which it is the idealized model. Growth is modeled by endowing the reference configuration of the shell with evolving first and second fundamental forms. Their evolution is such that the reference configuration remains a stress-free abstract Riemannian hypersurface. The evolution of the fundamental forms can be used to model both intrinsic (in-plane) bulk growth and remodeling (in the form of the evolution of the spontaneous curvatures). The fundamental forms are hence taken as dynamical variables in the Lagrange-dAlemberts principle for dissipative processes to derive both the governing equations of motion and the kinetic equations governing the evolution of the geometry of the reference shell configuration.

In the particular case when growth can be modeled by a Rayleigh dissipation potential, the kinetic equations take a form that couples the evolution of the fundamental forms with the stress state of the shell. As an example, starting from a flat planar sheet, different families of non-trivial geometries are found to be attainable by some stress-free growth processes.

Finally, the growth of a morphoelastic infinitely long cylinder subject to a time-dependent internal pressure and a radially symmetric non-uniform growth of an initially flat disk are investigated. The kinetic equations for growth along with the governing equations of motion are numerically solved to illustrate the coupling between the growth induced evolving geometry of the shell and the strain/stress state of the shell.