Modeling of the Suspension System of a Motorcycle

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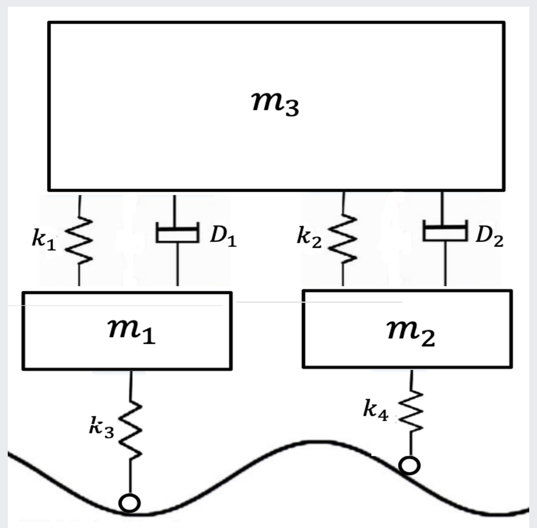
Introduction

A motorcycle suspension system is a complex system having multiple degrees of freedom. Suspensions consists of a system of springs, a dampener, and linkages that connects a chassis to wheels. Suspension systems minimize the effect of traveling on rough terrain, and provide a smooth and stable ride. The suspension is one of the most significant components of the motorcycle, so the performance motorcycles are directly affected by the characteristics of the suspension.

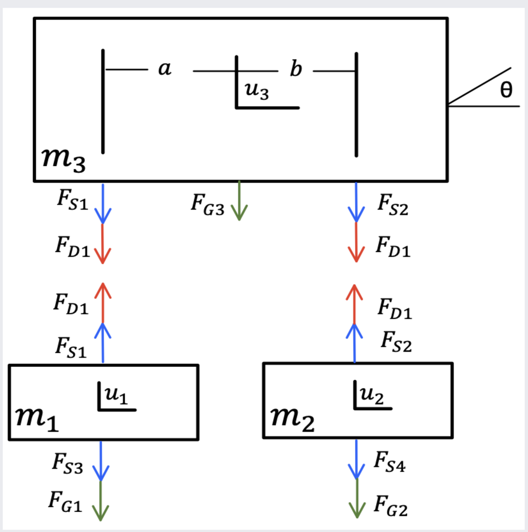
Each system's unique geometry and components allow them to function to specific user needs. For this project, the objective is to model and analyze a dual shock motorcycle suspension system. The input variables are spring constant, damping constant, and the mass of the chassis and wheels. Multiple solution curves will be generated using a fourth order Runge-Kutta method, Euler explicit method, Heun method, and the Collatz method.

Mathematical Modeling

A motorcycle suspension system is a complex system of springs and dampeners that control the reaction when encountering obstructions. The motorcycle is simplified down to three masses (frame and two wheels) being held by a spring dampener system, and modeling the tires as stiff springs.



Using the model from above the free body diagram can be described as the following:



Newtons Equations

Using a Newtonian approach, the equations motion for each mass are represented by the following equations:

$$\sum F = m\ddot{u}$$

$$m_1\ddot{u}_1 = F_{S1} + F_{D1} - F_{S3} - F_{G1}$$

$$m_2\ddot{u}_2 = F_{S2} + F_{D2} - F_{S4} - F_{G2}$$

$$m_3\ddot{u}_3 = -F_{S1} - F_{D1} - F_{S2} - F_{D2} - F_{G3}$$

$$\sum M = I\ddot{\theta} = F_{S1}a + F_{D1}a - F_{S2}b - F_{D2}b$$

The spring force is proportional to the displacement of each spring. These displacements relate to the position of the masses by the following:

$$x_1 = u_3 + u_1 - a\theta$$

$$x_2 = u_3 - u_2 + b\theta$$

$$x_3 = u_1 - u_s(t)$$

$$x_4 = u_2 - u_s\left(t + \frac{a+b}{v}\right)$$

Dampening force is proportional to the velocity. Differentiating the first two equations, an equation for the velocities are shown as follows:

$$\dot{x_1} = \dot{u_3} - \dot{u_1} - a\dot{\theta}$$

$$\dot{x_2} = \dot{u_3} - \dot{u_2} + b\dot{\theta}$$

Numerical Methods

Euler Explicit:

Euler explicit is one of the most common and simple numerical methods. Euler explicit typically introduces energy into a system and is nor accurate when the slope changes rapidly.

$$y_{j+1} = y_j + hf(t_j, y_j)$$

Heun Method

Heun Method is an improved version of the Euler explicit method. The method uses the midpoint slope any multiplies that slope by a step size to compute the next value.

$$y_{j+1} = y_j + \frac{h}{2}(f(t_j, y_j) + f(t_j + h, y_j + hf(t_j, y_j))$$

Collatz Method

The Collatz Method is an improved version of the Euler explicit method and will yield a solution curve similar to the Heun Method

$$y_{j+1} = y_j + hf(t_j + \frac{h}{2}, y_j + \frac{h}{2}f(t_j, y_j))$$

4^{th} **Order Runge-Kutta**

This method is the most accurate, and is defined by the following:

$$k_1 = f(x_j, y_j)$$

$$k_2 = f(x_j + \frac{1}{2}h, y_j + \frac{1}{2}k_1h)$$

$$k_3 = f(x_j + \frac{1}{2}h, y_j + \frac{1}{2}k_2h)$$

$$k_4 = f(x_j + h, y_j + k_3h)$$

$$y_{j+1} = y_j + \frac{1}{6}h(k_1 + 2k_2 + 2k_3 + k_4)$$

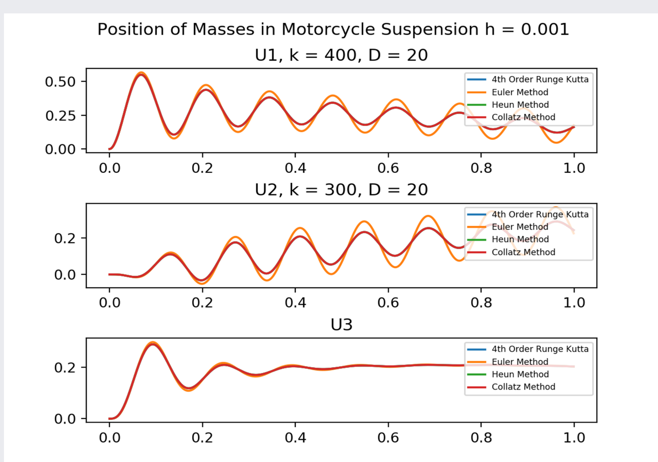
Results obtained from python program

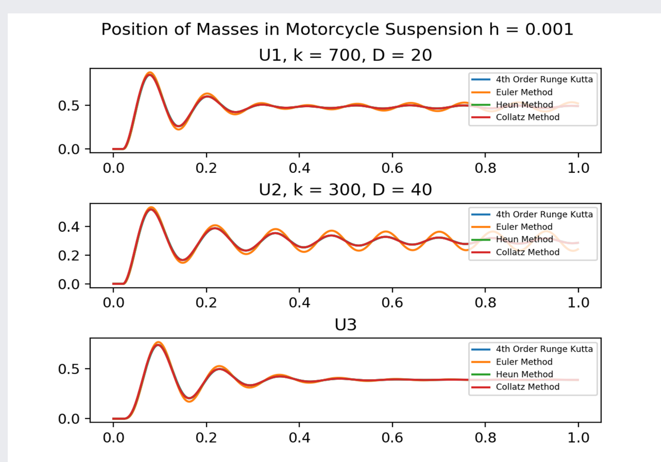
A python program was created in order to analyze the results. The optimal initial conditions are as follows:

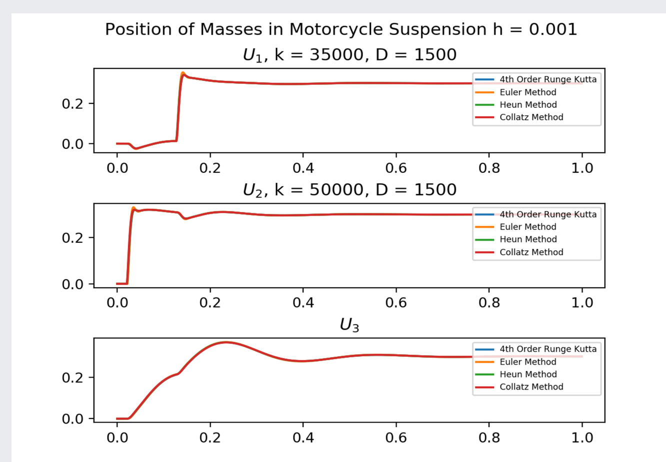
Initial Conditions:

- $a = 0.8$, $b = 0.8$
- $m_1 = 4$, $m_2 = 4$, $m_3 = 186$
- $D_1 = 2500$, $D_2 = 2500$
- $k_1 = 35000$, $k_2 = 95000$
- $k_3 = 455000$, $k_4 = 455000$
- $n = 1$, $h = 0.001$, $v_{road} = 15$

The graph on the left shows the resulting displacements with a sinusoidal function as the road. The top graph is the displacement of the rear of the motorcycle, the middle is the displacement of the front of the motorcycle, and the bottom is the displacement of the frame of the motorcycle. The last graph shows the resulting solution curve using the above parameters.







Conclusion

Overall, the goal of the project was to successfully model a motorcycle's suspension system. Another goal was to alter various parameters to learn how altering these parameters affects the results in order to design an optimized suspension system. The solution curve of the masses shows a sinusoidal curve that converges to a stabilization point.

Sources

References

[1] Multibody, Motorcycle Suspension (2016)