FRICTION LOSS ALONG A PIPE

1. INTRODUCTION

The frictional resistance to which fluid is subjected as it flows along a pipe results in a continuous loss of energy or total head of the fluid. Fig 1 illustrates this in a simple case; the difference in levels between piezometers A and B represents the total head loss h in the length of pipe l. In hydraulic engineering it is customary to refer to the rate of loss of total head along the pipe, dh/dl, by the term hydraulic gradient, denoted by the symbol i, so that

$$\frac{dh}{dl} = i$$

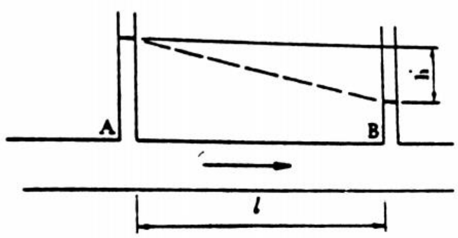

Fig 1 Diagram illustrating the hydraulic gradient

Osborne Reynolds, in 1883, recorded a number of experiments to determine the laws of resistance in pipes. By introducing a filament of dye into the flow of water along a glass pipe he showed the existence of two different types of motion. At low velocities the filament appeared as a straight line which passed down the whole length of the tube, indicating laminar flow. At higher velocities, the filament, after passing a little way along the tube, suddenly mixed with the surrounding water, indicating that the motion had now become turbulent.

Experiments with pipes of different and with water at different temperatures led Reynolds to conclude that the parameter which determines whether the flow shall be laminar or turbulent in any particular case is

$$R = \frac{\rho v D}{\mu}$$

In which

R denotes the Reynolds Number of the motion

ρ denotes the density of the fluid

v denotes the velocity of flow

D denotes the diameter of the pipe

μ denotes the coefficient of viscosity of the fluid.

The motion is laminar or turbulent according as the value to R is less than or greater than a critical value. If experiments are made with increasing rates of flow, this value of R depends degree of care which is taken to eliminate disturbances in the supply and along the pipe. On the hand, if experiments are made with decreasing flow, transition from turbulent to laminar place at a value of R which is very much less dependent on initial disturbances. This value of P. about 2000, and below this, the flow becomes laminar sufficiently downstream of any disturbance. matter how severe it is.

Different laws of resistance apply to laminar and to turbulent flow. For a given fluid flowing along a given pipe, experiments show that

for laminar motion I α V and 3

for turbulent motion I α V^n 4

n being an index which lies between 1.7 and 2.0 (depending on the value of R and on the roughness of the wail of the pipe) Equation 3 is in accordance with Poiseuille's equation which can be written in the form

i=$\frac{32\mu v}{\rho g D^2}$ 5

There is no similar simple result for turbulent now; in engineering practice it is custom Darcy's Equation

$$i = \frac{4fv^2}{D2g} \qquad \text{......6}$$

in which f denotes an experimentally determined friction factor which varies with R and pipe roughness.

The object of the present experiment is to demonstrate the change in the law of resistance and to establish the critical value of R. Measurements of i in the laminar region may be used to find the co-efficient of viscosity from equation 5 and measurements in the turbulent region may be used to find the friction factor f from equation 6.

2. DESCRIPTION OF APPARATUS

2. 1 . Overview

Fig 2 shows the arrangement in which water from a supply tank is led through a flexible hose to the bell-mouthed entrance to a straight tube along which the frictional loss is measured Piezometer tappings are made at an upstream section which lies approximately 45 tube diameters away from the pipe entrance, and at a downstream section which lies approximately 40 tube diameters away from the pipe exit. These clear lengths upstream and downstream of the test section are required to prevent the results from being affected by disturbances near the entrance and exit of the pipe. The piezometer tappings are connected to an inverted U-tube manometer, which reads the differential pressure directly in mm of water, or a U-tube which reads in mm of mercury.

The rate of flow along the pipe is controlled by a needle valve at the pipe exit, and is measured by timing the collection of water in a measuring cylinder (the discharge being so small as to make the use of the H1 Hydraulic Bench weighing tank impracticable)

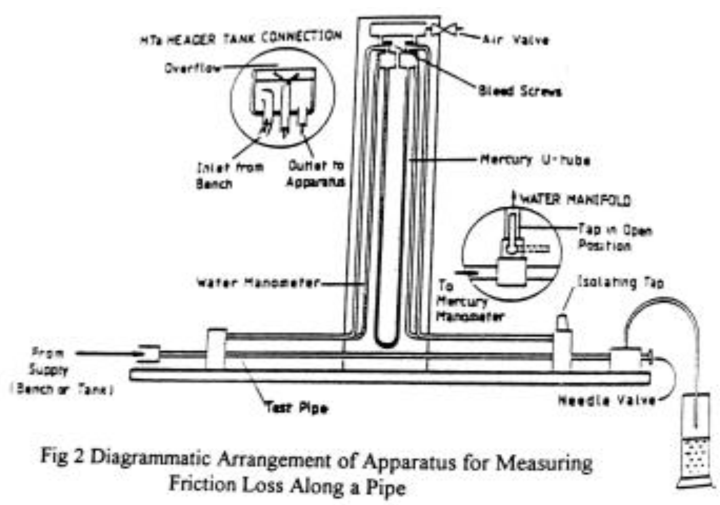

Fig 2 Diagrammatic Arrangement of Apparatus for Measuring Friction Loss Along a Pipe

2.<u>2 Installation and Preparation</u>

The apparatus is normally dispatched assembled and ready for use. In some instances, however, the manometer panel will be dismantled from the base-board of the apparatus. To reassemble:-

a) Secure the back panel supports to the baseplate with the two screws and washers provided.

These screws should not be excessively tightened.

b) Connect the free ends of the water and mercury manometer tubes to the pressure tapping block on the base board.

Secure these tubes with plastic ty-wrap clips using pliers to tighten them.

Superfluous lengths of ty-wrap should be cut off.

c) Assemble and connect the Header Tank, H7a, to the Hydraulic Bench supply and the inlet the 'friction in pipe' apparatus,

For higher flow rates, connect the plastic supply hose from the HI Hydraulic Bench directly to the inlet of the apparatus. Secure with the hose clip provided.

d) Connect the smaller bore plastic tube to the outlet port of the needle valve. Until measurements of flow are required, direct the free end of this tube into the access hole in the centre of the bench top.
For measurement direct the tube into a measuring flask. A litre flask (not supplied), sub-divided into 10 millilitre divisions, is most suitable.

e) Fill the U-tube manometer up to the 270 millimetres mark with mercury (not supplied). Approximately 40 millilitres will be required for this. Access ports are provided in the lower appropriate header.

f) Before allowing water to flow through the apparatus, check that the respective air purge valve and screw caps on the water and mercury manometer are both tightly closed.

CHECKING WATER MANOMETER CIRCUIT

A tap is provided at the downstream end of the test pipe for selecting either a water or mercury manometer circuit.

Avoid syphoning of the water when using the mercury manometer.

To check the circuit:-

a) Direct the tap towards the open position.

b) Allow a nominal flow of water through the apparatus.

Lightly tap the manometer tubes to clear air from the circuit.

c) Adjust the water levels in the tubes to the same height.

It may be necessary to connect a bicycle pump to the purge valve in the manifold and manipulate the levels accordingly.

d) Increase the water flow to obtain an approximate maximum scale reading. Observe these levels to ensure that they remain steady.

If there is a steady rise in the manometer levels, check that the valve is tightened and sealed properly.
If tightening does not stop the leak, replace the valve seal.
Check that the tube ferrules in the manifold are free from water blockage as this will suppress water levels and cause erroneous results. If this is suspected, a sharp burst of pressure from the bicycle pump will normally clean the blockage.

PURGING MERCURY MANOMETER

a) Turn the isolating tap to the Mercury Manometer circuit.

b) Purge all air from the manometer tubes by releasing the screw caps in the mercury manifold.

c) When purged, firmly screw down the manifold caps.

2.3 Routine Care and Maintenance

After use, the apparatus should be drained as far as possible and all external surfaces dried with a lint-free cloth.

Dry the Header Tank if this has been used. Care must be taken not to bend or damage the needle valve tip if this is removed. If the plastic manometer tubes become excessively discoloured a stain and deposit remover should be use.

THEORY OF FRICTION LOSS ALONG A PIPE

3.1 Derivation of Poiseuille's Equation

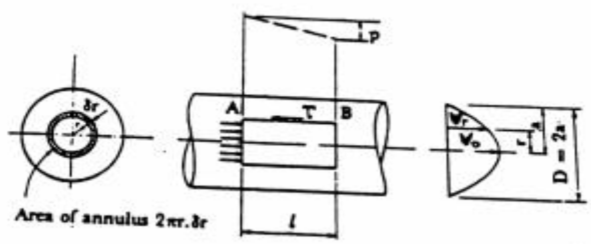

Fig 3 Derivation of Poiseuille's Equation

To derive Poiseuille's equation which applied to laminar flow along a tube, consider the motion indicated on Fig 3. Over each cross-section of the tube, the piezometric pressure is constant, and this pressure falls continuously along the tube. Suppose that between cross-sections A and B separated by length l of tube, the fall in pressure is p. Then the force exerted by this pressure difference on the ends of a cylinder having radius r, and its axis on the centre line of the tube, is $p\pi r^2$. Over any cross-section of the tube, the velocity varies with radius, having a maximum value of v_o the centre and falling to zero at the wall; let the velocity at radius r in any cross-section by denoted by v_r. Then the shear stress τ, in the direction shown on fig 3, due to viscous action on the curved surface of the cylinder, is given by

$$\tau = \frac{\mu d v_r}{dr}$$

(Note that $\frac{dv_r}{dr}$, is negative so that the stress acts in the direction shown in the figure). The force on the cylinder is due to this stress $\frac{\mu d v_r}{dr}.2\pi r l.$ Since the fluid is in steady motion under the action of the sum of pressure and viscous forces,

$$P.\pi r^2 + \frac{\mu d v_r}{dr} 2\pi r l = 0$$

Therefore $\frac{dv_r}{dr} = \frac{-pr}{2l\mu}$ 8

Integrating this and inserting a constant of integration such that

$v_r = 0$ when r = a

$V_r = \frac{p}{4l\mu}(a^2 - r^2)$ 9

This result shows that the velocity distribution across a section is parabolic, as indicated on fig 3, and that the velocity on the centre line, given by putting r = 0 in equation 9 is

$v_o = \frac{pa}{4l\mu}$ 10

The discharge rate Q may now be calculated. The flow rate through an annulus of radius r and width r is

$$\delta Q = V_r . 2\pi r \delta r$$

Inserting V_r from equation 9 and integrating

$$Q = \frac{p}{4l\mu} 2\pi \int_0^a (a^2 r - r^3) dr$$

Therefore $Q = \frac{p\pi a^4}{8l\mu}$ 11

Now the mean velocity V over the cross section is, by definition, given by

$$Q = v.\pi a^2$$

And elimimating Q between equation 11 and 12 gives

$V = \frac{pa^2}{8l\mu} = \frac{pD^2}{32l\mu}$ 13

By use of the substitution

$$\rho g h = p$$

And $\frac{h}{l} = i$

Eq. 13 may be written in the form

$$i = \frac{32\mu V}{\rho g D^2} \qquad \text{.............5}$$

(which is an equation of the form : y=mx+b)

3.2 Derivation of Darcy's Equation

If the flow is turbulent, the analysis given above is invalidated by the continuous mixing process which takes place. Across the curved surface of the cylinder having radius r in Fig 3, this mixing is manifest as a continuous unsteady and random flow into and out of the cylinder, so that the apparent shear stress on this surface is greater than the value given in equation 7. Because of the mixing, the distribution of velocity over a cross-section is more uniform than the parabolic shape deduced for laminar flow, as indicated on Fig 4.

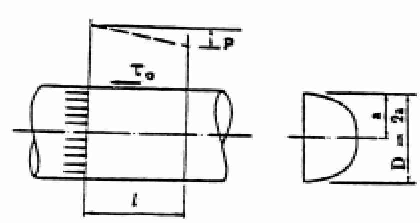

Fig 4 Derivation of Darcy's Equation

Although it is not possible to perform a complete analysis for turbulent flow, a useful result may be obtained by considering the whole cross-section as shown in Fig 4. It is reasonable to suppose that the shear stress τ_o on the wall of the tube will depend on the mean velocity v; let us assume for the present that

$$\tau_0 = f.\frac{1}{2\rho v^2}$$

In which $\frac{1}{2\rho v^2}$ denotes the dynamic pressure corresponding to the mean velocity v and f is a friction factor (not necessarily constant). Sinceτ_o and $1/2\rho v^2$ each have dimensions of force per unit area, f is dimensionless. The force on a cylinder of length l due to this stress id f.1/2pv^2.2πal, and the force due to the fall in pressure is p. πa^2 , so that

p. πa^2 = f. 1/2pv^2.2πal

Substituting

$$\rho g h = p$$

h/l = i

and a=$\frac{D}{2}$

leads to the result

i=$\frac{4f}{D}.\frac{v^2}{2g}$

which is form of Darcy's equation

The friction factor f which occurs in this equation was defined by equation 14 and is not necessarily constant. The results of many experiments show that f does, in fact, depend on both R, the Reynolds Number, and on the roughness of the pipe wall. At a given value of R, f increases with increasing surface roughness. For a given surface roughness, f generally decreases slowly with increasing R. This means that if R is increased by increasing v, so that the product fv^2 on which i depends equation 6 will increase somewhat less than v^2. In fact, over a fairly wide range, it is often possible, to represent the variation of i with v by the approximation

i = kv^n

where k and n are constants for a given fluid flowing along a given pipe, n having a value between 1.7 and 2.0.

4 **EXPERIMENTAL PROCEDURE**

4.1 Overview

The apparatus is set on the bench and leveled so that the manometers stand vertically. The water manometer is then introduced into the circuit by directing the lever on the tap towards the relevant connecting pipe. The bench supply valve is opened and adjusted until there is a steady flow down the supply tank overflow pipe. With the needle valve partly open to allow water to flow through the system, any trapped air is removed by manipulation of the flexible pipes. Particular care should be taken to clear the piezometer connections of air. The needle valve is then closed whereupon the levels in the two limbs of the inverted U-tube should settle to the same value. If they do not, check that flow has been stopped absolutely, and that all air bubbles have been cleared from the piezometer connections. The height of the water level in the manometer may be raised to a suitable value by allowing air to escape through the air valve at the top, or by pumping air through the valve.

Because of the large range of head differences involved, the readings are taken in two sets. Those for lower velocity flow rates, with the water manometer, and those for high velocity with the mercury manometer.

4.2 Water Manometer Readings

The needle valve is opened fully to obtain a differential head of at least 400 mm, and the collection of a suitable quantity of water in the measuring cylinder times. The values of h_1, (head in downstream manometer) and h_2 (head in upstream manometer) are now taken. Further readings may be taken at decreasing flows, the needle valve serving to reduce the discharge from each reading to the next. During this operation care should be taken: a) to ensure that the flow pipe exit is never below the surface of the water in the measuring cylinder; and b) to stand the measuring cylinder below the apparatus. Failure to observe these conditions will result in inaccurate flow rate readings, especially at the lower flow rates. The water temperature should be measured as accurately as possible at frequent intervals.

These readings should comfortably cover the whole of the laminar region and the transit turbulent flow; it is advisable to plot a graph of differential head against discharge as the experiment proceeds to ensure that sufficient readings have been taken to establish the slope of the straight line in the laminar region.

4.3 Mercury Manometer Readings.

The mercury manometer is now used, and the supply to the apparatus is taken directly from the bench supply valve instead of the elevated supply tank. Since the flexible hose between the bench supply valve and the apparatus will be subjected to the full pump pressure, it is advisable to secure the joints with hose clips.

Isolate the water manometer by turning the tap shown in Fig 2.

With the needle valve partially open and the pump running, the bench supply valve is opened fully. Air which may be trapped in the flexible hose is removed by manipulation, and bubbles in the piezometer connections arc induced to rise to the top of the U-tube, where they are expelled through bleed valves. There should then be continuous water connections from the piezometer tappings to the two surfaces of mercury in the U-tube and, when the needle valve is closed, the two surfaces should settle at the same level.

Readings of h_1 and h_2 are now taken starting with a maximum discharge and reducing in steps, the needle valve being used to set the desired flows. The water temperature should be recorded at frequent intervals.

It is desirable to take one or two readings at the lower end of the range which overlap the range already covered by the water manometer. Since a reading of 20 mm on the mercury U-tube corresponds to 252 mm on the water manometer, this requires one or two readings in the region 20 mm.

The diameter of the tube and the length between the piezometer tappings should be noted.

4.4 Procedures

Step 1: Record at least 8 sets of data over the range of the water manometer (see Section 4.2) and another 8 or more over the range of the mercury manometer (see Section 4.3). Tables 1 and 2 show the format of suitable result tables.

Results given in this section are typical of those obtainable from the equipment supplied. There will, however, be slight differences between individual units.

Step 2: Plot graphs of hydraulic gradient i against mean velocity v, and log i against log v. (Figs 6 and 7 show the form of graphs expected). (Reminder - the two manometers generate data for different operating ranges of the same system. The student must combine the data sets to analyze the system over the entire range.)

Step 3: From the (best fit) graph of i against log v, or graph of i vs v, determine the velocity at which rapid transition occurs. Determine the critical Reynold's Number at this velocity. (The student may elect to "blow up" that portion of the graph between 0.3 and 1.2 m/s)

Step 4: From the (best fit) slope of the graphs, derive the relationship between v and i. For both the upper and lower ranges, determine k and n where $i = kv^n$

Step 5: From the gradient of i against v in the laminar range, determine the coefficient of viscosity and compare with theoretical values.

Step 6: In the turbulent region of flow, select 4 or 5 values of velocity. Compute friction factors and Reynold's Number at these velocity values. Plot friction factors against Reynold's Number (Moody's Diagram) Compare with theoretical values.

5. TYPICAL RESULTS AND SAMPLE CALCULATIONS

5.1 Relationships between I and u

Length of pipe between piezometer tappings, l ………..524 mm

Nominal diameter of pipe, D ………..3 mm

Cross-sectional area of pipe, A ………..7.06 mm^2

Derivation of i over gauge length l

i) For water manometer

$i = \frac{(h_1 - h_2)}{l}$

ii) For mercury manometer

Referring to Fig 5, the specific gravity of mercury is taken as 13.6 writing the head difference in terms of water

$$i = \frac{(h_1 - h_2)(13.6 - 1)}{l}$$

Qty (ml)	t (s)	v (m/s)	h_1 (mm)	h_2 (mm)	h_1-h_2 (m)	i	θ $(°C)$	log i	log v

Table 1

Qty (ml)	t (s)	v (m/s)	h_1 (mm)	h_2 (mm)	h_1-h_2 (m)	i	θ $(°C)$	log i	log v

Table 2

Qty (ml)	t (s)	v (m/s)	h_1 (mm)	h_2 (mm)	h_1-h_2 (m)	i	θ ($°C$)	log i	log v
400	50.8	1.110	521.0	56.0	0.465	0.887	15.3	-0.0521	0.0453
400	54.0	1.049	500.0	85.0	0.415	0.794		-0.1002	0.0208
400	58.8	0.961	476.0	114.0	0.362	0.692		-0.1599	-0.0173
400	61.8	0.915	452.0	145.0	0.307	0.586		-0.2321	-0.0586
400	67.2	0.843	427.5	174.0	0.2535	0.483		-0.3161	-0.0742
300	57.8	0.734	390.0	223.0	0.167	0.319	15.3	-0.4962	-0.1343
300	71.9	0.592	375.0	245.0	0.130	0.248		-0.6055	-0.2277
300	92.9	0.457	362.0	263.0	0.099	0.189		-0.7235	-0.3401
200	92.4	0.306	349.0	282.0	0.067	0.128		-0.8928	-0.5143
150	100.8	0.220	340.0	295.5	0.455	0.085		-1.0771	-0.6576
85	113.6	0.106	332.5	306.0	0.0265	0.050	15.3	-1.2958	-0.9747
50	129.4	0.055	325.0	316.0	0.009	0.017		-1.7645	-1.2596

Table 1. Results with Water Manometer

Qty (ml)	t (s)	v (m/s)	h_1 (mm)	h_2 (mm)	h_1-h_2 (m)	i	θ ($°C$)	log i	log v
900	39.0	3.27	431.0	195.0	0.236	5.77		0.7612	0.5145
900	42.9	2.98	414.0	214.0	0.200	4.81		0.6821	0.4742
900	46.6	2.74	402.0	226.0	0.176	4.23		0.6263	0.4378
900	51.7	2.47	390.0	240.0	0.150	3.60	15.5	0.5563	0.3927
900	58.0	2.20	377.0	254.5	0.1225	2.94		0.4683	0.3424
900	62.7	2.03	370.5	261.1	0.1095	2.52		0.4014	0.3075
900	68.5	1.86	362.0	270.5	0.0915	2.20		0.3426	0.2695
600	47.5	1.77	358.5	275.0	0.0875	2.01		0.3052	0.2480
600	54.6	1.55	351.5	283.5	0.0680	1.69	15.9	0.2146	0.1903
600	70.4	1.19	340.0	294.0	0.0460	1.11		0.0434	0.0755
300	48.0	0.89	331.5	305.5	0.0360	0.87		-0.0625	-0.0531

Table 2. Results with Mercury U-tube

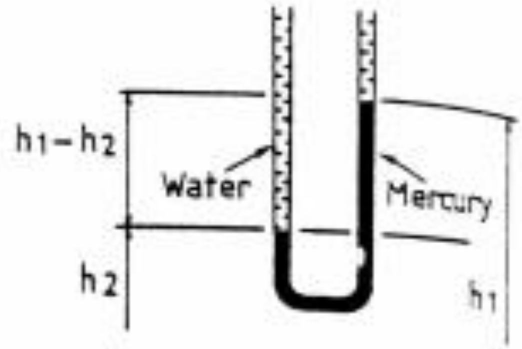

Fig.5 Diagram of U-tube

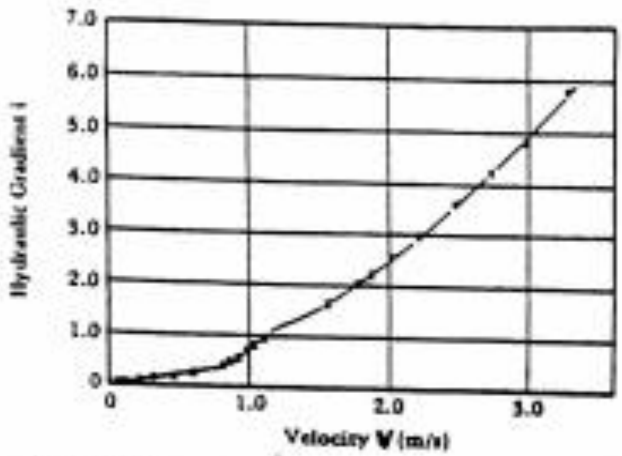

Fig. 6 (a). Variation of hydraulic gradient i with velocity V along pipe.

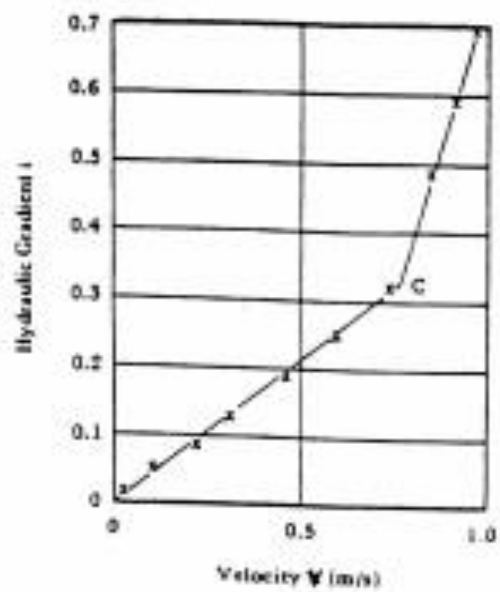

Fig. 6 (b). Variation of Hydraulic Gradient i with velocity V along pipe, up to 1.0 m/s.

From Fig 6a, graph of v against i, it can be seen that for small values of v, the frictional loss is proportional to velocity.

i.e. $i \propto v$

Fig 6b has been drawn with a larger scale for velocities up to 1 m/s.

This graph shows a fairly distinct change in the slope of the line at C when v is in the region of 0.77m/s. Up to this point the relationship is given by

i = 0.419 v (see section 5.3.1)

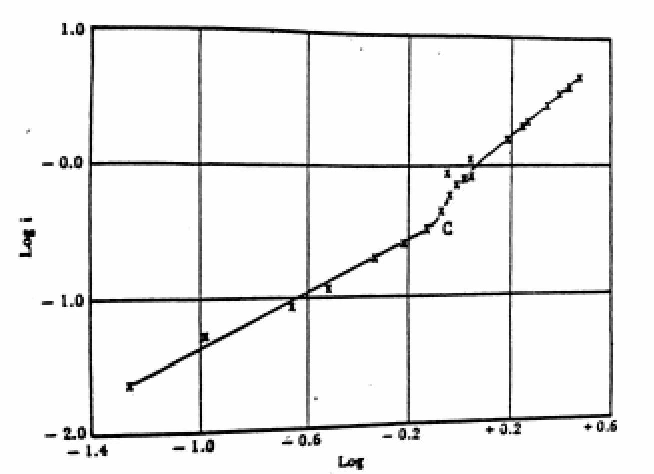

Fig 7 Variation of log i with log v for Flow Along Pipe

Point C marks the starts of a distinct transition phase where the flow characteristics change considerably.

In Fig.7 the same results are plotted to logarithmic scales.

Points up to C lie on a straight line of slope 1, confirming the frictional loss is proportional to velocity (See also Section 5.3.1)

For points above C we can write:-

$i \propto v^{1.69}$ for values of v greater than 1.5m/s (See also Section 5.3.2)

5.2 Calculations of Critical Reynolds Number

In Figures 6 and 7, Point C marks the distinct transition phase between laminar and turbulent flow. The velocity at Point C is approximately 0.77m/s

Recalling:- $R = \frac{\rho v D}{\mu}$ ----2

Substituting values we get at 15°C:

$$R = \frac{999X0.77X0.003}{11.4X10^{-4}}$$

= 2024

5.3 Calculation of Relationship between v and i

5.3.1 Laminar Range

$$i = kv^n$$

therefore, algebraically log i= log k + n log v

which is an equation of the forth y = mx+b

from Table 1, for v=306 and .592 (these points on best fit curve)

$n = \frac{\Delta \log i}{\Delta \log v} = \frac{.6055-(-.8928)}{.2277-(-.5143)} = \frac{.2873}{.2866} = 1.002$

say n = 1.00

log k = log i – n log v = -0.6055 – (1.00) (-.2277) = -.3778

k = .419

i = .419$v^{1.00}$

5.3.2 Turbulent Range

as noted above; log i = log k + n log v

from Table 2, for v = 1.55 and 2.47

$n = \frac{\Delta \log i}{\Delta \log v} = \frac{.5563-.2146}{.3927-.1903} = \frac{.3417}{.2024} = 1.688$

Say n = 1.69

log k = log i – n log v = .5563 – 1.69(.3927) = -.1073

k = 0.781

i = 0.781 $v^{1.69}$

5.4 Calculation of Coefficient of Viscosity

In the laminar range; i = $\frac{32\mu v}{\rho g D^2}$ (Eq. 5), which is an equation of the form y = mx + b

The slope of the plot is therefore = $\frac{32\mu}{\rho g D^2}$ Where slope = k = 0.419 (Section 5.3.1)

This can be rewritten in the form

$$\mu = k\frac{\rho g D^2}{32}$$

Substituting values we get

$$\mu = k\ \frac{\rho g D^2}{32}$$

Substituting values we get

$$\mu = \frac{0.419x999x9.81x9x10^{-6}}{32}$$

$\underline{\mu = 11.6\ x\ 10^{-4}\ \text{N.s/m}^2}$

5.5 Calculation of Friction Factor

In the turbulent region $i = \frac{4fv^2}{D2g}$ ------6

We can draw up table 3, giving values of f corresponding to various values of v, in the turbulent region of flow.

v(m/s)	I	$\frac{v^2}{2gD}$	f	R
1.6	1.75	43.7	0.0100	4260
2.2	3.00	82.2	0.0092	5860
2.8	4.45	132.8	0.0084	7450

Table 3 Calculation of the Friction Factor f in Darcy's Equation

6. DISCUSSION OF RESULTS

6.1 Measurements of frictional loss alone, the pipe at different velocities have shown two well-defined regions to which different laws of resistance apply. As the velocity is decreased from 3.3 to 1.5 m/s, frictional loss varied as v^{169}. Between 1.5 and 0.77, the loss decreased rather more steeply and as v decreased from 0.77 to zero, the loss varied directly as v. The critical velocity of 0.77 corresponds to a Reynolds number of 2024, this value being close to the figure of about 2000 at which transition from turbulent to laminar flow is usually found to take place.

6.2 The value of μ calculated by Poiseuille's equation applied to the results in the laminar region is

μ = 11. 6 x 10^{-4} Ns/m at 15.3°C.

The accepted value at this temperature is

μ =11.4 x 10^{-4} Ns/m^2

Since the accepted values are based on experiments with similar but more refined apparatus, the discrepance reveals an error of about 2% in the apparatus used here.

6.3 The results in the turbulent region have been used to calculate the friction factor f in Darcy's equation, and are found to fall with increasing v as shown in Table 4.